# ATTACHMENT A-1

# SOURCES AND ESTIMATION OF UNCERTAINTIES IN MEDICAL EXPOSURE

UNSCEAR 2020/2021 Report, Annex A, Evaluation of medical exposure to ionizing radiation

## Contents

This electronic attachment provides supplementary information on overall uncertainty determination in relation with total collective dose estimation from medical exposure.

## Acknowledgements

## Notes

This publication has not been formally edited.

# CONTENTS

# I.   TYPES OF ERROR

1.      In 2008, the report "Radiation Protection 154: European Guidance on Estimating Population Doses from Medical X-Ray Procedures" (RP 154) of the European Commission Dose Data Med project 1 (EC DDM 1) was published in the European Commission's Radiation Protection series. This report discussed in detail the estimation of uncertainties associated with medical exposure assessments and identified several important sources of uncertainty. In the following, frequent reference is made to RP 154 [E1].

2.      An estimate of the population dose, i.e., the total collective dose from all radiological examinations in a country, is based on frequency data and typical dose data for all examinations or categories of examinations considered. The overall uncertainty in the population dose estimate must, therefore, be a combination of the uncertainties in the estimates of the frequency and the uncertainties in the estimates of the effective dose for each type or category of examination.

3.      There are many potential sources of systematic error (or bias) and random (or statistical) error leading to uncertainties in frequency and dose estimates. In the first instance, all major sources of error are to be identified and evaluated in order to derive a measure of the overall uncertainty.

## A.   Systematic errors (bias)

4.      Systematic errors lead to estimates being systematically too high or too low. All observations or measurements of X-ray examination frequency and patient dose are prone to bias. Systematic errors can be due to insufficient knowledge or even complete lack of knowledge. Therefore, assumptions will have to be made that will not be completely valid. However, it is usually possible to make a rough assessment (educated guess) of the minimum ($a_{low}$) and maximum ($a_{upp}$) likely values of a quantity $a$ such that the probability that the value lies between these limits is, for all practical purposes, 100%. Provided that there is no contradictory information, the quantity can be treated as if it is equally probable for its value to lie anywhere within the interval [$a_{low}$, $a_{upp}$], corresponding to a uniform (i.e., rectangular) probability distribution. The best estimate of the value of the quantity is then:

$$a = \frac{a_{upp} - a_{low}}{2} \qquad\qquad (A\text{-}1.1)$$

with the standard uncertainty, $u$, being:

$$u = \frac{a}{\sqrt{3}} \qquad\qquad (A\text{-}1.2)$$

(e.g. [T2]).

## B.   Random (statistical) errors

5.      Random errors are present in all sampling procedures due to statistical fluctuations. They are equally likely to be positive or negative about the true value, usually following a normal distribution. They can arise from precision limitations in patient dose measurement devices or from surveys where data are collected from a sample of sites. Random errors can be reduced by increasing the number of observations.

If $\bar{N}$ is the arithmetic mean of $m$ values $N_i$,

$$\bar{N} = \frac{\sum_{i=1}^{m} N_i}{m},$$ 
(A-1.3)

the standard deviation, $s$, is given by

$$s = \sqrt{\frac{\sum_{i=1}^{m} (N_i - \bar{N})^2}{m-1}}$$
(A-1.4)

the standard uncertainty of the mean (also: "standard error of the mean SEM"), $u$, is then given by the standard deviation, $s$, divided by the square root of the number, $m$, of values $N_i$ (e.g., [T2])

$$u = \frac{s}{\sqrt{m}}$$
(A-1.5).

## II.   SOURCES OF ERROR IN FREQUENCY ESTIMATES

6.      There are basically two types of sources for deriving examination frequency data. Numbers are obtained either directly from a representative sample ("sample survey") of health care providers (e.g., hospitals) or from central statistics held by governmental departments or provided by health insurance companies ("insurance data survey"). In the case of a sample survey, an algorithm for scaling up to the whole country is needed. In the case of governmental central statistics or an insurance data survey, numbers are usually given for a large proportion or even the total of the population and might therefore be representative of radiology practice in the country without any need of scaling. Some countries apply a "mixed" approach, combining health insurance data with supplementary information from sample surveys [N1]. Insurance data from a representative sample of patients can also be used. In a study on population exposure from medical diagnostic procedures in France in 2012, the examination frequency was estimated from a permanent representative sample of the population protected by French health insurance, the Échantillon Généraliste de Bénéficiaires (EGB). The EGB is based on a survey of about 1% of the beneficiaries from the three main French health insurance schemes, whether these beneficiaries have received healthcare reimbursements or not (representing a coverage of around 85% of beneficiaries of all French social security schemes) [D1]. In the EC DDM 1 project [E1], some countries had sample sizes close to 100% (usually based on comprehensive national health insurance statistics) whereas others had to rely on limited surveys restricted to <20% of the total national practice.

7.      There are several sources of error which can lead to significant uncertainties in frequency estimates [E1]:

## A.  Incorrect counting due to problems in relating information (codes) into actual numbers of examinations

8.      Especially if health insurance data are used to estimate examination frequencies, there might be problems in assigning a specific code to a specific examination. Primarily, health insurance code systems are designed for reimbursement, and might therefore not be fully appropriate for a survey on radiological procedures. There might, for example, be insufficiently differentiated codes, that is, single codes which refer to X-ray examinations of different sites (e.g., only one code for knee joint and shoulder joint).

9.      It is important to have an adequate definition of the term "examination", which is to be used consistently. The EC DDM 1 project [E1] definition of an examination was as follows: "An X-ray examination or procedure is defined as one or more (a series of) X-ray exposures of one anatomical region/organ/organ system using a single imaging modality, needed to answer a specific diagnostic problem or clinical question during one visit to the radiology department or medical practice", i.e. one radiological examination may consist, for example, of:

–   Several X-ray images (e.g., radiographs of thoracic spine in anteroposterior and lateral projections);

–   One X-ray image combined with some fluoroscopy (e.g., plain radiography of chest organs with fluoroscopy);

–   A prone plus a supine computer tomography (e.g., colonography);

–   A nuclear medicine procedure plus a computer tomography scan (e.g., hybrid imaging);

–   A dual energy computer tomography scan involving either two X-ray sources or fast kV switching technique.

These examples are to be counted as one examination. It should be established whether the code system accounts for examinations or for single steps of an examination in order to avoid any bias.

10.     Codes often cover only a limited number of examination types and therefore some radiological procedures are not taken into account within this framework, although these are assumed to represent only a small fraction of the total number. The most explicit example is that of procedures performed under fluoroscopy guidance in operating theatres. Estimating the frequency of these procedures is very complicated. Even the detailed categorization of examinations used in the EC DDM 1 project [E1], with 225 types of examinations in total, omitted these procedures.

11.      In the case of X-ray examinations of double-sided organs, e.g., mammograms, coding systems do not always distinguish properly between examinations involving one side (usually used for examinations of symptomatic persons) or both sides (usually used for screening), respectively. In this case, it might be difficult to estimate accurate numbers.

12.      For a "multistep" examination, the corresponding dose must be thoroughly assessed. This is especially important in the case of higher dose procedures like computer tomography where it can make a big difference depending on whether the number of examinations or the number of individual scans is counted, and whether the dose per examination or the dose per scan, respectively, is used.

13.      In a sample survey where data are received from larger healthcare providers like hospitals, data on frequency should be available from the Radiology Information System (RIS) and/or Hospital Information System (HIS). In RISs and HISs, predefined code systems are used, but many different coding systems exist (even in the same country). Uncertainties in counting can, therefore, be introduced through insufficient information on the coding system available. Thus, modifications in code systems need to be taken into account and periodic reviews of the coding system would therefore be necessary.

### B.   Mistakes in the data recorded, e.g., no assessment of duplicate examinations or typing errors

14.      Repeated examinations, e.g., due to overexposure, underexposure, and position fault, will in some instances not be documented. Especially in the case of health insurance data, repeated examinations are usually not recorded and a bias can, therefore, be introduced. Similarly, in a sample survey where data are collected from a picture archive and communications system (PACS), it might also be the case that the PACS is set automatically to remove images from the reject folder after a short time, e.g., a few days.

15.      The repeat rate is dependent on the type of examination, type of technique (conventional/digital) and practitioner experience [N2, W1]. On the basis of a survey of international papers on reject/retake rates published both for screen/film and digital technology, Waaler et al. concluded that the use of digital imaging seems to have reduced the percentage of image rejects/retakes from 10–15% to 3–5% mainly due to a striking reduction of images with over-/underexposure [W1]. An educated guess should be considered to account for this kind of error.

16.      Mistakes due to typing errors are generally of a random nature and are difficult to estimate. It can, however, be expected that they might cancel out.

## C.  Lack of frequency data from some important providers of radiology services

17.     In publication RP 154 [E1], a list of healthcare providers is given that might be involved in providing radiology services (table A-1.1). Their inclusion in national surveys can be used as a first step in assessing the completeness of the frequency data in each country. However, missing data from some of the health care providers given in table A-1.1 should not be critical since their contribution to total frequency and population exposure is minor (school dental services, health checks at borders, prisons, armed forces hospitals/units and medical research exposures).

**Table A-1.1. Healthcare providers involved with X-ray imaging [E1]**

| Healthcare providers | |
|---|---|
| University hospitals | Dental practices |
| State hospitals | Dental institutes |
| Private hospitals | School dental services |
| Private radiology institutes | Health checks at borders |
| General practices | Prisons |
| Specialist practices (e.g. cardiology, gastroenterology, orthopaedics, pneumology, urology, vascular surgery) | Tuberculosis screening units |
| Occupational medicine | Breast cancer screening units |
| Chiropractic clinics | Armed forces hospitals/units |

18.     Key providers not covered by central statistics might be, e.g., private radiology practices, or practices offering opportunistic screening examinations. The contribution of examinations from the latter to the total frequency is likely to vary from country to country, where industrialized countries are more prone to radiology services offering "off-label" imaging procedures. The potential bias due to missing these examinations should be assessed.

19.     A major contributor to frequency data is dental radiology, accounting for about one third or even more of all X-ray examinations in industrialized countries [E1, E2]. However, missing this component will have only a minor impact on the total collective dose.

## D.  Unrepresentative sample

20.     In a sample survey, the samples should be as representative of national radiology practice as possible, especially if frequency data are derived from a relatively small sample. All health care providers that might offer medical radiology services should be considered (see table A-1.1) and the main contributors to national radiology practice should be accounted for. Here, the aim should be to include in the sample all types of hospital and radiological practice (or the most important ones) in similar proportions to those occurring nationally, i.e., avoid some categories of

radiology being more likely to be included in the sample than others (and a biased, i.e. non-random sample). Self-selection might occur if institutes volunteer to be included in the sample and are not representative in view, for example, of their better than average quality management system.

### E.   Invalid assumptions made when scaling up data to derive frequencies for the whole country

21.    Numbers of examinations observed in a sample or incomplete data from a health insurance survey must be scaled up to the whole country. However, uncertainties will be introduced wherever the assumption is made that the distribution of examinations seen in the sample is the same as that for the overall population. The method of scaling up is dependent on the availability of specifically detailed data. A stratification process can be performed dividing the overall radiology practice in a country into subgroups, e.g., groups of radiology service providers, or groups of patients. Sampling in each stratum will usually reduce sampling error and improve the representativeness of the sample as a whole. The better the representativeness of the sample, the larger will be the reliability of the frequency estimate.

## III.   UNCERTAINTIES IN FREQUENCY ESTIMATES

### A.   Uncertainty in frequency measurements by counting

22.    Although count is a common quantity, the issue of uncertainty in counting has received little consideration. Thus, it is usually assumed that such measurements, the concept of uncertainty is not applicable, or even, that a result derived by counting actually has no uncertainty at all [B1]. The latter might be intuitively true if small numbers are assessed by counting, e.g., the number of persons in a small room. However, if the number of persons in a crowded marketplace is to be counted, the result will probably be affected by an uncertainty arising from double counts, $n_+$, or missing counts, $n_-$. The "true" value, $M$, of the counting result, $N$, is then:

$$M = N - n_+ + n_- \qquad \text{(A-1.6)}$$

If the probability of a double count is $p_+$, i.e., $n_+ = p_+ \cdot M$, and the probability of a missing count is $p_-$, i.e., $n_- = p_- \cdot M$, then $M$ would be

$$M = \frac{N}{1 + p_+ - p_-} \qquad \text{(A-1.7)}$$

$N$ can be assumed to be the sum of independent counts, $n_i$, i.e.

$$N = \sum_{i=1}^{M} n_i \qquad \text{(A-1.8)}$$

where each count, $n_i$, can have the value 0 (with probability $p_-$), 1 (with probability 1 - ($p_+ + p_-$)), or 2 (with probability $p_+$). The error probability is ($p_+ + p_-$) since $p_+$ and $p_-$ are disjoint events, and the probability of correct counting is 1 - ($p_+ + p_-$) [B1].

The variance, $v$, of $n_i$ is then

$$v(n_i) = p_+ + p_- - (p_+ - p_-)^2 \qquad \text{(A-1.9)}$$

and the variance, $v$, of $N$ is

$$v(N) = M \cdot (p_+ + p_- - (p_+ - p_-)^2) \qquad \text{(A-1.10)}$$

The corresponding uncertainty, $u$, associated with $M$ would, thus, be [B1]

$$u = \frac{\sqrt{M \cdot (p_+ + p_- - (p_+ - p_-)^2)}}{1 + p_+ - p_-} \qquad \text{(A-1.11)}.$$

If it can be assumed that the probabilities of double counts and missed counts are the same, $p_+ = p_-$, the measurement by counting, $N$, is equal to the true value, $M$, and the uncertainty is small for large $N$ (e.g., $u = 0.3\%$ for $N = 10{,}000$ and $p_+ = p_- = 5\%$).

23.     If there is no reason to believe that $p_+$ and $p_-$ are high and/or differ widely, the uncertainty in frequency measurement by counting is, therefore, minor; in particular, for larger $N$, as is usually the case for frequencies of radiological examinations, it can be assumed to be negligible.

## B.   Uncertainty in frequency estimate for "sample survey"

24.     As an example, consider the methodology of the United Kingdom survey by the National Radiation Protection Board [T1]. This survey was based on data from two National Health Service (NHS) regions in 1997/98. A sample of 38 NHS hospital trusts in these regions provided data from their RIS with detailed information on annual numbers of different types of X-ray examination. Feedback was received from 58% of all trusts covering 68% of all X-ray examinations in the two regions, and about 16% of all X-ray examinations in England. The survey data were extrapolated to the whole of the NHS by means of annual total numbers of all types of X-ray examination conducted in each trust that were collected by the Department of Health. The same approach was chosen to estimate X-ray examination frequencies in NHS hospitals in Wales and Northern Ireland. Since identical levels of radiology provision were found in England, Wales and Northern Ireland, the same level was assumed to apply to Scotland where no total numbers were available.

25.     For each type of X-ray examination, the standard deviation in the percentage frequency at each trust was calculated. The standard deviation, $s$, was then converted to the standard error of the mean, $u$, to account for the variations between the $m$ institutions in the sample for each type of X-ray exam:

$$u = \frac{s}{\sqrt{m}} \qquad \text{(A-1.12)}$$

the SEM, $u$, decreases with increasing number of institutions, $m$.

## C.   Uncertainty in frequency estimate for "insurance data survey"

26.     If the survey is based on health insurance data (or otherwise data from central statistics), data are usually available at regular (e.g., annual) intervals. In this case,

time series of frequency data can be derived. The assessment of time series of frequencies enables one to detect and analyse discontinuities or outliers by comparing figures from different years. If discontinuities can be explained, e.g., through a change in the code system, the data can be revised, and the overall uncertainty in the population dose estimate can therefore be decreased. In the case of an inexplicable discontinuity, typing errors or similar might be present, and the outlier can be considered for exclusion.

27.    Even if health insurance data can be assumed to be complete, random errors should nevertheless be taken into account. Having time series of frequencies, the standard deviation, s, can be calculated. If *m* is the number of points of time where frequency data were evaluated, and $N_y$ are the frequency estimates, *s* is defined as the square root of the variance or the square root of the mean square deviations of the values $N_y$, from their arithmetic mean, $\bar{N}$:

$$s = \sqrt{\frac{\sum_{y=1}^{m}(N_y - \bar{N})^2}{m-1}} \; with \; \bar{N} = \frac{\sum_{y=1}^{m} N_y}{m} \tag{A-1.13}.$$

28.    The standard deviation may serve as a measure of uncertainty. Therefore, assuming that variations in the X-ray examination frequencies over a certain period of time are interpreted as random uncertainties, the uncertainties in $N_y$ can be approximated by the standard deviation of the corresponding *m* frequencies, $N_y$. This would be a conservative approach. If the time series shows clear evidence of a trend, this can be accounted for via a regression analysis by using the "residual standard deviation", $s_r$, which can be written as

$$s_r = \sqrt{\frac{\sum_{y=1}^{m}(N_y - \tilde{N}_y)^2}{m-p}} \tag{A-1.14}$$

where $\tilde{N}_y$ is the value on the regression curve and *p* is the number of parameters in the regression function (e.g. *p* = 2 in case of a linear regression). The uncertainty at a 95% confidence level is about twice the (residual) standard deviation (figure A-1.I).

**Figure A-1.I. Example of time series of examination frequencies with decreasing trend**

The residual standard deviation is 145,000. The uncertainty in the 2015 frequency value at the 95% confidence level is about twice the residual standard deviation or 10% of the 2015 frequency value
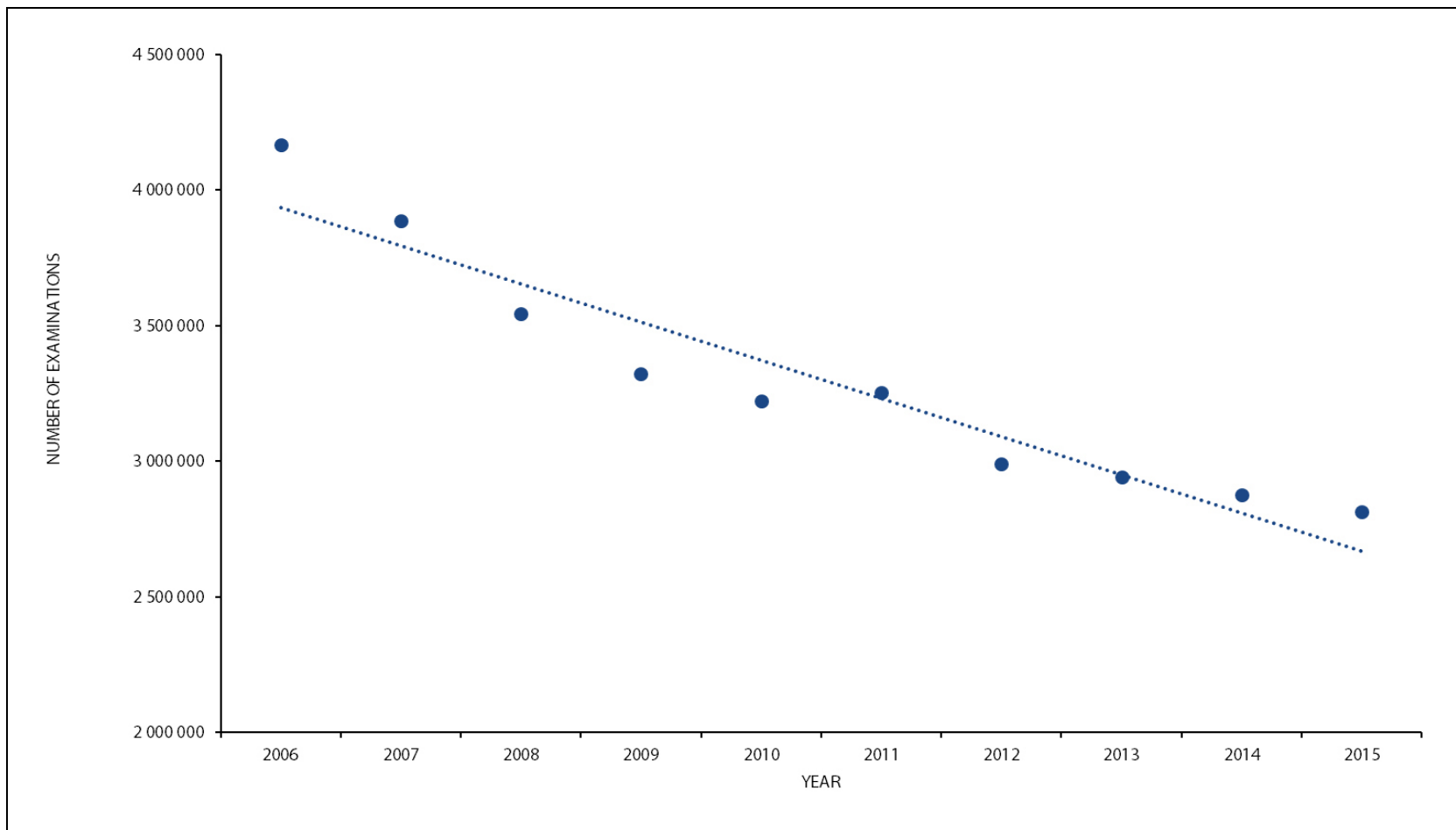
# IV.  UNCERTAINTIES IN DOSE ESTIMATES

29.     In order to estimate the population dose from medical exposure, estimates of representative mean effective doses for each type of X-ray or nuclear medicine examination (or of those that make a significant contribution to the collective dose) are needed. Typically, patient dose surveys are performed at a limited number of imaging sites. In the case of X-rays, such surveys are based on measurements or calculations of practical dose quantities (e.g., entrance surface dose (ESD); dose-area product (DAP)). However, doses delivered in a specific institution for a standard clinical indication can be fraught with a high variability in data collected within and between patient samples. As an example of the application of statistical considerations in the selection of an appropriate sample size in local surveys, see, for example, Taylor et al. who determined the variability of computed tomography dose index (CTDI) and dose-length product (DLP) data, and proposed a minimum sample size to achieve an expected precision [T3]. For nuclear medicine procedures, doses per procedure are derived by multiplying the mean administered activity with a dose coefficient which relates the activity to the effective dose to the patient, as published by the International Commission on Radiological Protection (ICRP) [I1, I2, I3, I4]. Administered activities can vary considerably between imaging sites.

30.     To derive a typical national value for a particular type of examination, mean doses from a sample of representative imaging sites are usually collected. For each site, mean dose values (or mean administered activities for nuclear medicine procedures) for every type of examination (category) are calculated, and mean values of the "site means" are assessed. Uncertainties in effective dose estimates can arise from a range of different sources [E1].

## A.   Uncertainties in basic dose measurements

31.     Generally, uncertainties in dose measurement are small compared with the variation in dose seen in a sample of patients undergoing the same X-ray examination in the same hospital and compared with the variation in mean doses for the same X-ray examination between all hospitals in a national survey.

32.     In accordance with Report 74 [I5] of the International Commission on Radiation Units and Measurements (ICRU), an uncertainty of no more than 7% at the 95% confidence level is, in general, achievable for patient dose measurements in diagnostic radiology if the calibration procedures and measurement methods are taken into account as described in the ICRU report [I5]. Since this cannot be assumed for all patient dose surveys, the uncertainties might be higher. The uncertainties in individual basic dose measurements are, nevertheless, small compared with the uncertainties due to the variation in dose seen in a sample of patients undergoing the same X-ray examination in the same institution, and small compared with the variation in mean doses for the same X-ray examination between all institutions in a national survey. Consequently, they will not have a significant impact on the overall accuracy of the mean effective dose estimate associated with each type of X-ray examination.

33.    There are numerous sources of uncertainty in estimates of collective effective dose in nuclear medicine in relation to both the number of procedures and the effective dose from those procedures. A significant number of common nuclear medicine examinations can be performed using different protocols which may involve several different radiopharmaceuticals (e.g., myocardial perfusion scans can be performed using a 1-day protocol, 2-day protocol, $^{201}$Tl-Cl, $^{99m}$Tc-sestamibi or $^{99m}$Tc-tetrofosmin). The effective dose from each of these protocols may vary significantly and national statistics may not clearly distinguish between the different protocols.

34.    Internal dose estimations are based on mathematical phantoms derived from computer tomography data of the human anatomy and biokinetic parameters related to the behaviour of the particular radiopharmaceutical, namely the fractional uptake and effective half-time in organs of the body. Stabin [S1] has shown that the combined uncertainties in any given radiopharmaceutical dose estimate are typically, at a minimum, a factor of two and may be considerably greater, in general because of normal human variability, and particularly in disease states.

## B.   Uncertainties due to variations in patient dose between imaging sites and limited sample size of sites

35.    As a source for estimating random uncertainties, the dose distributions observed in the UK National Patient Dose Database—one of the most extensive databases of this type in Europe—can be utilised. Uncertainties were derived from the observed variation of the mean dose values at each imaging site.

36.    The standard deviation, $s$, was converted to the standard error of the mean, $u$, to account for the variations between the $m$ institutions in the sample for each type of X-ray exam,

$$u = \frac{s}{\sqrt{m}} \qquad\qquad (A\text{-}1.15)$$

with $s$ being the standard deviation of the site mean doses for each type of X-ray examination. Since the SEM, $u$, is decreasing with increasing number of institutions, $m$, values of uncertainty in the estimated mean dose value at the 95% confidence level (2 * SEM) can be approximated as a function of sample size, as shown in table A-1.2 (based on table 17 in RP 154 [E1]).

**Table A-1.2. Uncertainties in mean effective dose estimates as a function of sample size, based on UK National Patient Dose Database (from [E1])**

| Sample size (number of imaging sites) | Uncertainties at 95% confidence level (%) |
|---|---|
| >100 | ±10 |
| 20–100 | ±25 |
| 5–19 | ±50 |

37.     These estimates refer merely to random uncertainties, i.e. any systematic errors due to bias in a sample of imaging sites are not accounted for. If no dose measurements have been performed for a particular examination, an estimate of mean effective dose from another country can be assumed. Optimally, a country is selected where the radiology practice is expected to be similar to that in the home country and the estimate is based on measurements at least 20 imaging sites. If this is not the case, the uncertainties will be larger. In RP 154 [E1], a factor of 2 is suggested for the uncertainty at a 95% confidence level (+100%, −50%).

## C.   Uncertainties due to the conversion coefficients used to assess effective doses

38.     Uncertainties due to conversion coefficients depend on how closely the exposure conditions and the phantom for which the conversion coefficients were calculated match the average exposure conditions and the average patient for a specific X-ray examination. These systematic uncertainties are difficult to quantify but should be small for common examinations (with a 95% confidence limit of probably no more than about ±10%). For less common examinations, the uncertainty estimates could rise to about ±25%.

39.     Another consideration is the common practice of applying a single value of typical effective dose in relation to both adult and paediatric patients. Whereas there are significant differences in the typical levels of practical dose metrics between patients of different (age-related) standard size, this is offset by corresponding differences in the effective dose coefficient such that the variations in effective dose between adults and children undergoing similar types of examination are reduced [E1]. For most types of examination, adults also account for the majority of patient numbers. However, use of a single value of E does represent a potential source of uncertainty in estimates of population dose.

## D.   Overall uncertainties in mean effective dose estimates

40.     On the basis of the considerations in subsections IV.A to IV.C, the following estimates, shown in table A-1.3, for uncertainties in dose estimates were proposed by EC DDM 1 project as a rough guide [E1].

**Table A-1.3. Overall uncertainties in mean effective dose estimates as a function of sample size and matching of exposure conditions for conversion coefficients [E1]**

| Sample size (number of imaging sites) and matching of conversion coefficients | Uncertainties at 95% confidence level (%) | | |
|---|---|---|---|
| | Sample size | Conversion coefficient | Overall |
| >100 Good conversion coefficients match | ±10 | ±10 | ±14 |
| 20–100 Good conversion coefficients match | ±25 | ±10 | ±27 |
| 5–19 Good conversion coefficients match | ±50 | ±10 | ±51 |
| >100 Poor conversion coefficients match | ±10 | ±25 | ±27 |
| 20–100 Poor CC match | ±25 | ±25 | ±35 |
| 5–19 Poor conversion coefficients match | ±50 | ±25 | ±56 |
| Foreign data only[a] | | | +100 −50 |

[a] Unless the radiology practice in the foreign country is similar to that in the country making the estimation.

## V.  OVERALL UNCERTAINTIES – PROPAGATION OF UNCERTAINTY

41.     The result of a survey on medical population exposures cannot be assessed directly but is composed of several numbers and measures from different sources that are combined following a predefined algorithm.

42.     Since most of these quantities are associated with random or systematic errors, the final result of a survey will also deviate from the true value, i.e. each error will be transferred by the algorithm that is to be performed. This is called propagation of uncertainty and there exist established procedures to estimate the total deviation of the final result from the true value (e.g. [T2]).

43.     Assuming the sources of error to be independent (not correlated), the overall standard uncertainty $u$, of the quantity $N$ which is a function of other quantities $N_1, N_2....N_x$, is given by

$$u = u(N_1, ..., N_x, u_1, ..., u_x) = \sqrt{\sum_{i=1}^{x} \left( \frac{\partial N}{\partial N_i} u_i \right)^2} \qquad \text{(A-1.16)}$$

where $u_j$ is the absolute uncertainty of $N_j$ and $\frac{\partial N}{\partial N_i}$ is the partial derivative of $N$ with respect to $N_i$.

44.     If the quantity of interest, $N$, is made up of a sum,

$$N = \sum_{i=1}^{x} N_i \qquad \text{(A-1.17)}$$

the above formula is reduced to

$$u = \sqrt{\sum_{i=1}^{x} u_i{}^2} \qquad\qquad\qquad (A\text{-}1.18)$$

i.e. $u$ is the root mean square sum of the absolute uncertainties $u_i$.

If $N$ is made up of a product,

$$N = \prod_{i=1}^{x} N_i \qquad\qquad\qquad (A\text{-}1.19)$$

the above given formula is reduced to

$$u = N \sqrt{\sum_{i=1}^{x} \left(\frac{u_i}{N_i}\right)^2} \qquad\qquad\qquad (A\text{-}1.20)$$

i.e. the relative uncertainty, $u/N$, is the root mean square sum of the relative uncertainties of $N_i$. See figure A-1.II for an example of a product with two factors.

45.     Commonly, there are errors affecting the result in one direction as well as errors affecting the results in the other direction. Therefore, there is a certain chance that these errors cancel out. This is especially true for random errors. Therefore, even some large individual uncertainties might have only a small impact on the overall uncertainty. However, systematic errors can, at worst, also add up.

46.     To exclude in this case a serious under- or over-estimation of the overall uncertainty, uncertainties with a clear sign $\pm$ should be considered separately. For example, in the case of repeat examinations, the error will have a plus-sign. To avoid an underestimation of the overall frequency and also an overestimation of the overall uncertainty, the repeat rate, $r$, should already be included in the frequency estimate, $M$,

$$M = N + r\,N \qquad\qquad\qquad (A\text{-}1.21)$$

with $N$ being the frequency value observed and $r$ being the repeat rate not accounted for in $N$. The associated uncertainties, $u_r$, and $u_N$, can then be allowed for separately to estimate the uncertainty $u_M$ of $M$:

$$u_M = M \sqrt{\left(\frac{u_N}{N}\right)^2 + \left(\frac{u_r\,N}{M}\right)^2} \qquad\qquad\qquad (A\text{-}1.22)$$

47.     To illustrate the propagation of uncertainties, two examples are given:

  *(a)* Example 1: Table A-1.4 illustrates the propagation of uncertainty in relation to the sum of five examination counts, $n_a$ to $n_e$, each associated with a specific level of uncertainty, $u_a$ to $u_e$. The total absolute uncertainty is given by:

$$u = \sqrt{\sum_{i=A}^{E} u_i{}^2} \qquad\qquad\qquad (A\text{-}1.23)$$

  The uncertainties for examinations C–E are relatively high, but their individual contributions to the overall uncertainty are minor since these examinations collectively account for only 15% of the total number of examinations.

**Table A-1.4. Example of estimating uncertainties from the number of examinations (frequencies)**

| Examinations | Absolute number of examinations | 95% confidence level | |
|:---:|:---:|:---:|:---:|
| | | Absolute uncertainty on frequency | Relative uncertainty on frequency (%) due to variance |
| A | 7 000 000 | 420 000 | 6 |
| B | 4 500 000 | 360 000 | 8 |
| C | 1 400 000 | 210 000 | 15 |
| D | 350 000 | 35 000 | 10 |
| E | 260 000 | 65 000 | 25 |
| **Total** | **13 510 000[a]** | **596 280[b]** | **4.4** |

[a] Sum of absolute numbers of frequencies.
[b] Root mean square sum of absolute uncertainties on frequency.

    *(b)* Example 2: Table A-1.5 presents the combination of frequencies, $N_A$ to $N_E$, and estimates of effective dose per examination type, $E_A$ to $E_E$, to assess the total collective dose, $S$.

$$S = \sum_{i=A}^{E} N_i \; E_i \qquad (A\text{-}1.24)$$

For each examination A-E, the relative uncertainty $v_i$ in the collective dose $N_i \cdot E_i$ is estimated by the root mean square sum of the relative uncertainty in the frequency estimate, $u_i/N_i$, and the relative uncertainty in the estimate of mean effective dose per examination, $u_{Ei}/E_i$ (i=A to E).

$$v_i = \sqrt{\left(\frac{u_i}{N_i}\right)^2 + \left(\frac{u_{E_i}}{E_i}\right)^2} \qquad (A\text{-}1.25)$$

Since the relative uncertainty $v_i$, in example 2, is considerably larger than the relative uncertainty $u_i/N_i$ for the frequency, the effective dose uncertainty per examination is the determining factor for the value of the collective dose uncertainty (figure A-1.II).

**Figure A-1.II. Propagation of Uncertainty - Uncertainty of a Product**

**Uncertainty of $N \cdot M$ (z-axis) where $N$ and $M$ are associated with relative uncertainties <100% (x- and y-axes)**

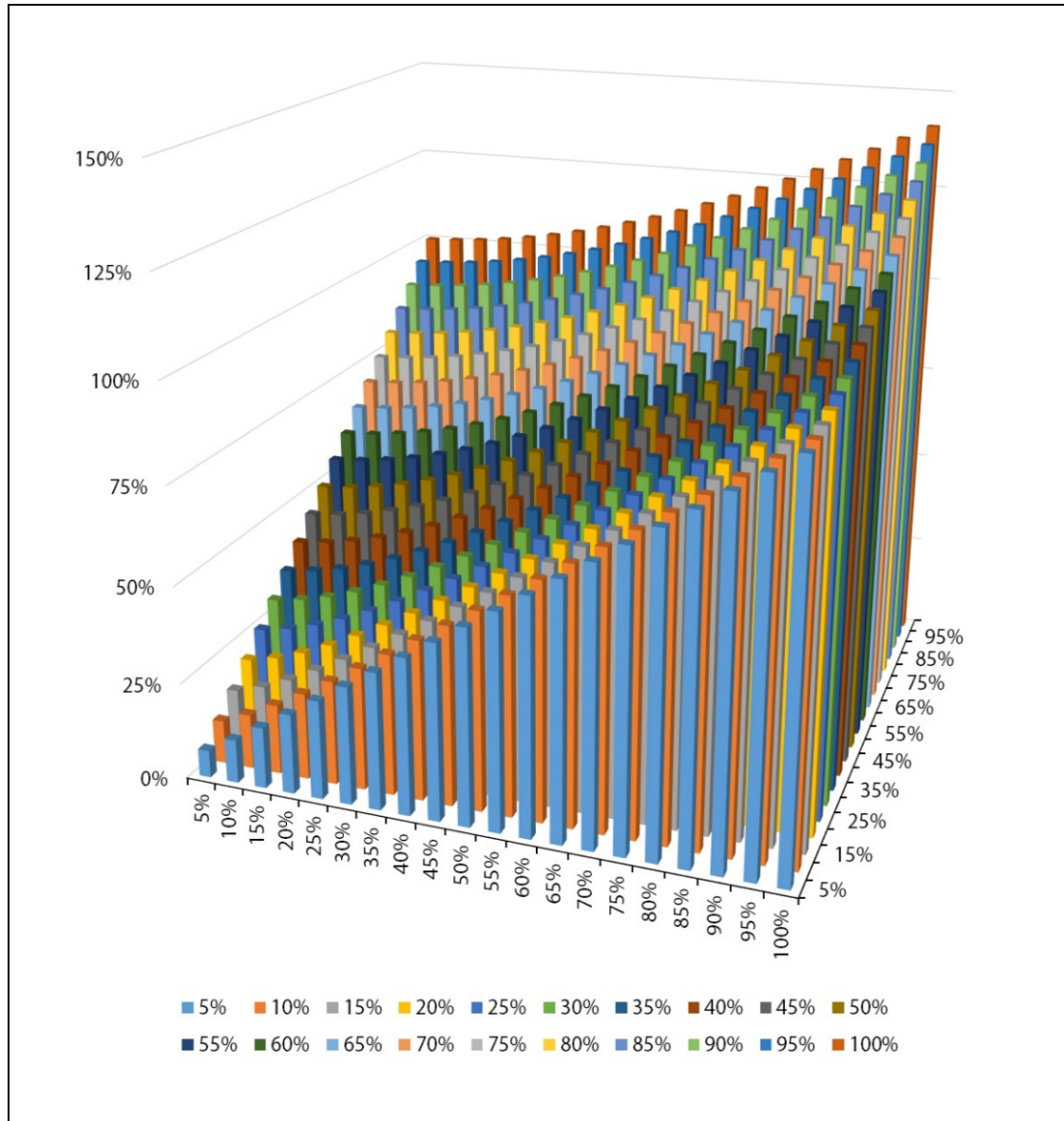The maximum of the $N \cdot M$ uncertainty is $\sqrt{2}$ (141%)

**Table A-1.5. Examples of estimating uncertainties for total collective effective dose**

| Examinations | Absolute number of examinations | Relative uncertainty on frequency[a] (%) | Effective dose per examination (mSv) | Relative uncertainty on effective dose per examination (%)[a] | Collective effective dose (man Sv) | Absolute uncertainty on collective effective dose (man Sv) | Relative uncertainty on collective dose[b] (%) |
|---|---|---|---|---|---|---|---|
| A | 7 000 000 | 6 | 0.3 | 15 | 2 100 | 339 | 16 |
| B | 4 500 000 | 8 | 0.1 | 35 | 450 | 161 | 36 |
| C | 1 400 000 | 15 | 0.4 | 50 | 560 | 292 | 52 |
| D | 350 000 | 10 | 2.2 | 20 | 770 | 172 | 22 |
| E | 260 000 | 25 | 0.6 | 80 | 156 | 130 | 84 |
| **Total** | **13 510 000** | **4.4** | | | **4 036[c]** | **522[d]** | **13** |

[a] 95% confidence level.
[b] Root mean square sum of relative uncertainty on frequency and relative uncertainty on dose estimate.
[c] Sum of collective doses.
[d] Root mean square sum of absolute uncertainties on collective dose.

48.     The impact of examination A's uncertainty in association with the collective dose for examination A is decisive since A contributes most to total collective effective dose. The impact of examination E, on the other hand, is, in spite of the large uncertainty, minor since E contributes least to total collective effective dose. Zontar et al. [Z1] provides an example of the assessment of the overall uncertainty for a national estimate of collective effective dose.

49.     It can be assumed that the combined standard uncertainty takes the form of a normal distribution and there is a 68% confidence that the estimated value lies within the stated limits. If uncertainties are expressed in terms of the 95% confidence limits, this corresponds approximately to twice the standard uncertainties.

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